

# INDIAN MARITIME UNIVERSITY

(A Central University Government of India)

## END SEMESTER EXAMINATION DECEMBER 2017

**Programme** : B. Tech (Marine Engineering) **Semester** : II  
**Subject Name** : Mathematics II **Subject Code** :  
UG11T2202/UG11T1202  
**Date** : 19.12.2017 **Maximum Marks** : 100  
**Time** : 3 Hrs. **Pass Marks** : 50

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### PART A

(All questions are compulsory)

Marks 10 x 3 = 30

1. a) Find  $a_0$  in the expansion of  $f(x)$  as Fourier series in the interval  $(-2, 2)$ , where

$$\begin{aligned} f(x) &= 0 & -2 \leq x \leq 0 \\ &= 1 & 0 \leq x \leq 2 \end{aligned}$$

- b) Find  $b_n$  in the expansion of  $f(x)$  as Fourier series in the interval  $(-\pi, \pi)$  if

$$\begin{aligned} f(x) &= -1 & -\pi < x \leq -\pi/2 \\ &= 0 & -\pi/2 < x < \pi/2 \end{aligned}$$

$$= 1 \quad \pi/2 < x < \pi$$

- c) Find the Laplace transform of  $F(t) = 1 \quad 0 < t \leq 1$   
 $= t \quad 1 < t \leq 2$   
 $= 0 \quad t > 2$

- d) Find the inverse Laplace transform of

$$\frac{3(s^2 - 2)^2}{2s^5}$$

- e) Solve  $\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = 0$  given  $y(0) = 0$ ;  $\frac{dy}{dx} = 15$  at  $x = 0$

- f) Find the particular integral for the differential equation

$$(D^2 + 4D + 4)y = 3 \sin x + 4 \cos x$$

- g) Solve  $\frac{dy}{dx} - \frac{dx}{dy} = \frac{x}{y} - \frac{y}{x}$

- h) A random variable  $x$  has the following probability function

$x$	-2	-1	0	1	2	3
$f(x)$	0.1	k	0.2	2k	0.3	k

Find the value of  $K$  and calculate mean and variance of the distribution.

- i) If on an average 1 vessel in every 10 is wrecked, find the probability that out of 5 vessels expected to arrive, at least 4 will arrive safely.

j) A variate  $x$  has the probability distribution

$x$	-3	6	9
$p(x)$	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{3}$

Find  $E(x)$ ,  $E(x^2)$

### PART B

Answer any 5 from the following

14 x 5 = 70

2. a) Find the Fourier series expansion of  $f(x) = 2x - x^2$  in  $(0,3)$  and hence deduce that

$$\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} \dots - \infty = \frac{\pi^2}{12} \quad (14 \text{ marks})$$

3. a) Find the Laplace transform of

$$t \int_0^t \frac{e^{-t} \sin t}{t} dt$$

- b) Find the inverse Laplace transform of  $\tan^{-1} \left( \frac{2}{s^2} \right)$  (6 + 8 marks)

4. a) Using convolution theorem, find  $L^{-1} \left\{ \frac{1}{(s+a)(s+b)} \right\}$

- b) Solve the following differential equation using Laplace transform

$$y''' + 2y'' - y' - 2y = 0 \text{ given } y(0) = y'(0) = 0 \text{ and } y''(0) = 6$$

(6 + 8 marks)

5. a) Solve  $(xy^3 + y)dx + 2(x^2y^2 + x + y^4)dy = 0$

- b) If the stream lines (paths of fluid particles) of a flow around a corner are  $xy = \text{constant}$ , find their orthogonal trajectories (called equipotential lines)

(7 + 7 marks)

6. a) Solve  $(D^2 - 2D + 1)y = xe^x \sin x$

- b) Solve by variation of parameters the following differential equation

$$\frac{d^2y}{dx^2} + 4y = \tan x$$

(7 + 7 marks)

7. a) In a test, an examinee either guesses, or copies or knows the answer to Multiple choice questions with four choices. The probability that he makes a guess is  $\frac{1}{3}$  and the probability that his answer is  $\frac{1}{6}$ . The probability that his answer is correct, given that he copies is  $\frac{1}{8}$ . Find the probability that he knew the answer to the question given that he correctly answered.

- b) Find the probability distribution of the number of green balls drawn when 3 balls are drawn one by one without a replacement from a bag containing three greens and five white balls.

(8 + 6 marks)

8. a) The probability density  $p(x)$  of a continuous random variable is given by

$$p(x) = y_0 e^{-|x|} - \infty < x < \infty$$

Prove that  $y_0 = \frac{1}{2}$ . Also find the mean and variance of the distribution.

- b) A die is tossed thrice, A success is getting 1 or 6 on a toss. Find the mean and variance of the number of success.

(7 + 7 marks)

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